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Student Life

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Arithmetic and geometric sequence and series

Study Development Worksheet

Questions

1. The first three terms in an arithmetic sequence are 1, 7 and 13. Find the common difference.

2. The second, third, and fourth terms in an arithmetic sequence as 6, 8 and 10. What is the first term and the common difference?

3. In an arithmetic sequence, the first three terms are $a_1 = 5y$, $a_2 = 15y^2 - 5$ and $a_3 = 15$. If $a_1 > 0$, write an expression for a_n.

4. The first three terms in a geometric sequence are 3, 6 and 12. What is the common ratio of the sequence?

5. The third, fourth, and fifth terms in a geometric sequence are 2, 4 and 8. What is the first term and the common difference?

6. The first three terms of a geometric sequence are $a_1 = y$, $a_2 = y^2 - 3$ and $a_3 = \frac{1}{y}$. Given that $a_1 > 0$, and $a_3 < 0.6$, find a suitable value for y and the common ratio r.

7. A ball is bounced on the floor. After the first bounce, the ball reaches a maximum height of 2m. After each bounce, the height that the ball reaches decreases by 30%.
   a. What height does the ball reach after 6 bounces?
   b. At what point does the height of the ball go below 70cm?
   c. What is the sum to infinity of the height of the bounces?

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Answers

1. The general formula for the terms in an arithmetic sequence is $a_n = a_1 + (n-1)d$ where d is the common difference. We begin by writing out the terms in this format:

$$a_1 = 1$$
$$a_2 = a_1 + (2-1)d = 1 + d = 7$$
$$a_3 = 1 + 2d = 13$$

Therefore, using a_2, we can see that $d = 7 - 1 = 6$.

2. We write out the terms using the general formula:

$$a_2 = a_1 + d = 6$$
$$a_3 = a_1 + 2d = 8$$
$$a_4 = a_1 + 3d = 10$$

We find an expression for d by finding $a_3 - a_2 = a_1 + 2d - a_1 - d = 8 - 6$ which gives us $d = 2$. We then use a_2 to find $a_2 = a_1 + 2 = 6$ which gives us $a_1 = 4$.
Therefore, an expression for a_n is given by $a_n = 4 + (n-1)d$.

3. We write out the terms using the general formula:

$$a_1 = 5y$$
$$a_2 = a_1 + d = 15y^2 - 5$$
$$a_3 = a_1 + 2d = 15$$

We then find two expressions for d in terms of y:

$$a_2 - a_1 = a_1 + d - a_1 = d = 15y^2 - 5 - 5y$$
$$a_3 - a_2 = a_1 + 2d - a_1 - d = d = 15 - 15y^2 + 5$$

We then equate the two expressions for d:

$$15y^2 - 5 - 5y = 15 - 15y^2 + 5$$

Which we rearrange to get a quadratic:

$$30y^2 - 5y - 25 = 0$$

Since there is a common factor of 5 in each term, we divide through by 5:

$$6y^2 - y - 5 = 0$$

Which we factorise to get

$$(6y + 5)(y - 1) = 0$$

Which gives us two possible values of y: $y_1 = -\frac{5}{6}$ and $y_2 = 1$.

When we use y_1 to find a_1, we get $a_1 = 5\left(-\frac{5}{6}\right) = -\frac{5}{3}$, which contradicts the question that states that $a_1 > 0$, so we know that $y = 1,$ since this gives us $a_1 = 5(1) = 5$.

Therefore, $d = 15y^2 - 5 - 5y = 15(1)^2 - 5 - 5(1) = 5$, so the expression for a_n is given by $a_n = 5 + 5(n - 1)$.

4. The general formula for a term in a geometric sequence is $a_n = ar^{n-1}$. We write out each term using this formula:

$$a_1 = ar^0 = a = 3$$
$$a_2 = ar^1 = ar = 6$$
$$a_3 = ar^2 = 12$$

We then find $\frac{a_2}{a_1} = \frac{ar}{a} = r = \frac{6}{3} = 2$. Therefore, an expression for $a_n = 3 \times 2^{n-1}$.

5. We write out the terms using the general formula:

$$a_3 = ar^2 = 2$$
$$a_4 = ar^3 = 4$$
$$a_5 = ar^4 = 8$$

We then find an expression for r by finding $\frac{a_4}{a_3} = \frac{ar^3}{ar^2} = r = \frac{4}{2} = 2$.

We use this to find a value for a by $a_3 = ar^2 = a(2)^2 = 4a = 2$ which gives $a = \frac{1}{2}$.

Therefore, we can find an expression for a_n: $a_n = \frac{1}{2} \times 2^{n-1}$.

6. We write out the terms using the general formula:

$$a_1 = a = y$$

$$a_2 = ar = y^2 - 3$$

$$a_3 = ar^2 = \frac{1}{y}$$

We then find two expressions for r in terms of y:

$$\frac{a_2}{a_1} = \frac{ar}{a} = r = \frac{y^2 - 3}{y}$$

$$\frac{a_3}{a_2} = \frac{ar^2}{ar} = r = \frac{\left(\frac{1}{y}\right)}{y^2 - 3} = \frac{1}{y(y^2 - 3)}$$

We then equate the two expressions:

$$\frac{y^2 - 3}{y} = \frac{1}{y(y^2 - 3)}$$

We cross multiply to get:

$$y(y^2 - 3)(y^2 - 3) = y$$

Then divide both sides by y:

$$(y^2 - 3)(y^2 - 3) = 1$$

We expand and rearrange:

$$y^4 - 6y^2 + 8 = 0$$

Then we factorise to get:

$$(y^2 - 4)(y^2 - 2) = 0$$

So, our potential values for y are $2, -2, \sqrt{2}, -\sqrt{2}$.

By the conditions in the question, we need $a_1 = y$ to be greater than 0, which means y cannot be -2 or $-\sqrt{2}$.

We also need y to give us $a_3 < 0.6$. Since $\frac{1}{\sqrt{2}} \approx 0.707$, we know that $y \neq \sqrt{2}$.

Therefore, $y = 2$.

We then find r:

$$r = \frac{1}{y(y^2 - 3)} = \frac{1}{2(2^2 - 3)} = \frac{1}{2}$$

7. This question is eluding to us writing a geometric sequence formula for the height of the bounce. Since the height of the bounce is decreasing by 30% each time, we know that each bounce height is 70% of the bounce height before it. So, each bounce height is 0.7 times the height of the bounce before it. This is, therefore, our common ratio. We have our first term, as the first bounce is 2m high. We use the general formula to give an expression for the height after n bounces:

$$a_n = 2 \times 0.7^{n-1}$$

a. $a_6 = 2 \times 0.7^5 = 0.33614\text{m}.$

b. We find how many bounces the ball has gone through when the height is exactly 0.7m:

$$0.7 = 2 \times 0.7^{n-1}$$

Which we rearrange to get:

$$0.7^{n-1} = \frac{0.7}{2} = 0.35$$

We use a logarithm function to find:

$$n - 1 = log_{0.7}(0.35) = 2.943$$

Since we can't have 2.943 bounces, we round to the next highest whole number, which is 3.

c. Since $r < 1$, we can use this formula to find the sum to infinity of the heights:

$$\sum_{k=0}^{\infty} a \times r^k = a\frac{1}{1 - r}$$

$$\sum_{k=0}^{\infty} 2 \times 0.7^k = 2\left(\frac{1}{1 - 0.7}\right) = 6.\dot{6}\text{m}$$

Note: If you have not seen the logarithm function before, there are some worksheets on it in the maths success section.

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