Quadratic Equations

Study Development Quick Guide

A quadratic equation is a polynomial of the form

$$f(x) = ax^2 + bx + c$$

where a, b, c are constants and $a \neq 0$.

In order to solve (find x such that $f(x) = 0$) a quadratic equation, we can use one of several methods:

1. **Factorising by inspection**

   This is where we put the equation in the form $(px + q)(rx + s) = 0$, such that $pq = a$, $ps + qr = b$, and $qs = c$. Sometimes this is quite easy to figure out, especially when $a = 1$. In this case, we tend to ask the question ‘what two numbers add to give b and multiply to give c? The solutions to the quadratic equation are then given by $x = \frac{-q}{p}$ and $x = \frac{-s}{r}$.

2. **The quadratic formula**

   This is a foolproof method for solving. If you are ever struggling to factorise a quadratic, this should be your go-to method.

   We find values for x when $ax^2 + bx + c = 0$ by evaluating:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   Note: $\pm$ in this case means ‘do the calculation once with a + where the $\pm$ is, and then once with a – where the $\pm$ is’

3. Completing the square

We complete the square on $ax^2 + bx + c = 0$ by carrying out the following steps:

- Divide by a to get: $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
- Take $\frac{c}{a}$ away from both sides to get: $x^2 + \frac{b}{a}x = -\frac{c}{a}$
- Rewrite the left-hand side in the format: $\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}$
- Add $\frac{b^2}{4a^2}$ to both sides to get: $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$
- Take the square root of both sides: $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$
- Take $\frac{b}{2a}$ away from both sides to give a final answer of: $x = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$

This method can be a bit complicated or difficult to remember. If it makes sense to you, then use it, as it works for all quadratic formulae, however we generally recommend that you use the quadratic formula instead as it's a bit quicker.

Support: Study Development offers workshops, short courses, 1 to 1 and small group tutorials.

- Join a tutorial or workshop on the Study Development tutorial and workshop webpage or search 'YSJ study development tutorials.'
- Access our Study Success resources on the Study Success webpage or search 'YSJ study success.'