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Logarithms

Study Development Factsheet

What is the logarithm function?

How would you solve $4^x = 64$ to get a value for x?

Perhaps you have seen this calculation before, and so you know that the answer is x = 3. Often, however, we are asked to solve functions that have less obvious solutions, such as finding y when $2^y = 33{,}554{,}432$.

We could go through each power of 2 until we find the correct answer, but this is very time consuming, and in a lot of cases x or y will not be integers.

This is why we have a logarithm function.

In order to calculate c when $a^c = b$, we use logs:

$log_a(b) = c$.

So, in our earlier examples, we have that $log_4(64) = 3$, and $log_2(33{,}554{,}432) = 25$.

Calculating logs

We read the function $log_a(b) = c$ as ‘log base a of b equals c’.

On a scientific or graphical calculator, there will often be three buttons to allow you to calculate logs. There is usually a $\log_{10}$ button, a $\log_x$ button, and a natural log button called ln(more on this later).

In order to calculate a log, you can use a calculator, or simplify it using log laws.

Natural logs

The natural log function is the inverse of the exponential function. If we were to calculate x when $e^x = 10$, we would use our log function and calculate $log_e(10) = x$.

This function, 'log base e' is called the natural log function, and we write it as $\ln$. So, we would calculate $log_e(10) = ln(10) = x$.

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Log laws

There are several rules that the logarithm function follows. It is worth learning these as you will likely be required to use them often.

- $log_a(b) + log_a(c) = log_a(b \times c)$
- $\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$
- $b\,log_a(c) = log_a(c^b)$
- $log_a(a) = 1$
- $log_a(1) = 0$
- $a^{log_a(b)} = b$
- $log_a(a^b) = b$
- $ln(e^x) = x$

Examples

1. Simplify $log_2(4) + log_2(8)$.
2. Calculate $3\,log_3(1) - log_3(3)$.
3. Calculate $log_7(49)$.

Answers

1. $log_2(4) + log_2(8) = log_2(4 \times 8) = log_2(32) = 5$.

   Alternatively, $log_2(4) + log_2(8) = 2 + 3 = 5$.

2. $3\,log_3(1) - log_3(3) = log_3(1^3) - log_3(3) = log_3(1) - log_3(3) = log_3\left(\frac{1}{3}\right) = -1$.

   Alternatively, $3\,log_3(1) - log_3(3) = 3 \times 0 - 1 = -1$.

3. $log_7(49) = log_7(7^2) = 2$

   Alternatively, $log_7(49) = 2$.

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