YORK
ST JOHN
UNIVERSITY
Est. 1841

Student Life
Library and
Learning Services

Rationalising the denominator and making it real
Study Development Worksheet

Rationalising the denominator

Example

Rationalise the denominator of $\frac{2}{\sqrt{2}}$.

Answer

$$\frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\left(\sqrt{2}\right)^2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Questions

Rationalise the denominator of:

1. $\frac{1}{\sqrt{3}}$

2. $\frac{3}{2\sqrt{5}}$

3. $\frac{2}{2 - \sqrt{3}}$

4. $\frac{5}{\sqrt{7} + 4}$

5. $\frac{10}{4\sqrt{2} + 9}$

Library and Learning Services

Study Development

Email: studydevelopment@yorksj.ac.uk

Est. 1841

YORK
ST JOHN
UNIVERSITY

YORK ST JOHN UNIVERSITY
Est. 1841
Student Life
Library and Learning Services

Rationalising the denominator and making it real
Study Development Worksheet

Answers

1. $\dfrac{1}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{\left(\sqrt{3}\right)^2}$

$= \dfrac{\sqrt{3}}{3}.$

2. $\dfrac{3}{2\sqrt{5}} \times \dfrac{2\sqrt{5}}{2\sqrt{5}} = \dfrac{6\sqrt{5}}{\left(2\sqrt{5}\right)^2} = \dfrac{6\sqrt{5}}{4\times5} = \dfrac{6\sqrt{5}}{20}$

$= \dfrac{3\sqrt{5}}{10}.$

3. $\dfrac{2}{2-\sqrt{3}} \times \dfrac{-\sqrt{3}-2}{-\sqrt{3}-2} = \dfrac{2(-\sqrt{3}-2)}{(2-\sqrt{3})(-\sqrt{3}-2)} = \dfrac{-2\sqrt{3}-4}{-2\sqrt{3}-4+\left(-\sqrt{3}\right)^2+2\sqrt{3}} = \dfrac{-4-2\sqrt{3}}{-4+3} = \dfrac{-4-2\sqrt{3}}{-1}$

$= 4 + 2\sqrt{3}.$

4. $\dfrac{5}{\sqrt{7}+4} \times \dfrac{\sqrt{7}-4}{\sqrt{7}-4} = \dfrac{5(\sqrt{7}-4)}{(\sqrt{7}+4)(\sqrt{7}-4)} = \dfrac{5\sqrt{7}-20}{\left(\sqrt{7}\right)^2+4\sqrt{7}-4\sqrt{7}-16} = \dfrac{5\sqrt{7}-20}{7-16} = \dfrac{5\sqrt{7}-20}{-9}$

$= \dfrac{20-5\sqrt{7}}{9}$

5. $\dfrac{10}{4\sqrt{2}+9} \times \dfrac{4\sqrt{2}-9}{4\sqrt{2}-9} = \dfrac{10(4\sqrt{2}-9)}{(4\sqrt{2}+9)(4\sqrt{2}-9)} = \dfrac{40\sqrt{2}-90}{\left(4\sqrt{2}\right)^2+36\sqrt{2}-36\sqrt{2}-81} = \dfrac{40\sqrt{2}-90}{32-81}$

$= \dfrac{40\sqrt{2}-90}{-49}$

Making the denominator real

Example

Write $\dfrac{10}{4+3i}$ in the form a + bi.

Answer

$$\frac{10}{4+3i} \times \frac{3i-4}{3i-4} = \frac{10(3i-4)}{(4+3i)(3i-4)} = \frac{30i-40}{12i-16+9i^2+12i} = \frac{-40+30i}{-16-9} = \frac{-40+30i}{-25}$$

$$= \frac{-8+6i}{-5} = \frac{8}{5} - \frac{6}{5}i \ .$$

Remember:

Questions

Write the following in the form a + bi:

1. $\dfrac{1}{5i}$

2. $\dfrac{-5}{2i-1}$

3. $\dfrac{-37}{3+\frac{i}{2}}$

4. $\dfrac{1-3i}{1-2i}$

5. $\dfrac{i}{5+4i}$

YORK
ST JOHN
UNIVERSITY
Est. 1841

Student Life
Library and
Learning Services

Rationalising the denominator and making it real
Study Development Worksheet

Answers

1. $\dfrac{1}{5i} \times \dfrac{5i}{5i} = \dfrac{5i}{(5i)^2} = \dfrac{5i}{25i^2} = \dfrac{5i}{-25}$

   $= \dfrac{-i}{5}.$

2. $\dfrac{-5}{2i-1} \times \dfrac{2i+1}{2i+1} = \dfrac{-5(2i+1)}{(2i-1)(2i+1)} = \dfrac{-5-10i}{(2i)^2 - 2i + 2i - 1} = \dfrac{-5-10i}{4i^2 - 1} = \dfrac{-5-10i}{-4-1} = \dfrac{-5-10i}{-5}$

   $= 1+2i.$

3. $\dfrac{-37}{3+\frac{i}{2}} \times \dfrac{\frac{i}{2}-3}{\frac{i}{2}-3} = \dfrac{-37\left(\frac{i}{2}-3\right)}{\left(3+\frac{i}{2}\right)\left(\frac{i}{2}-3\right)} = \dfrac{-37\left(\frac{i}{2}-3\right)}{\frac{3i}{2} - 9 + \frac{i^2}{4} - \frac{3i}{2}} = \dfrac{-37\left(\frac{i}{2}-3\right)}{-9 - \frac{1}{4}} = \dfrac{-37\left(\frac{i}{2}-3\right)}{\frac{-37}{4}} = 4\left(-3 + \dfrac{i}{2}\right)$

   $= -12+2i$

4. $\dfrac{1-3i}{1-2i} \times \dfrac{-2i-1}{-2i-1} = \dfrac{(1-3i)(-2i-1)}{(1-2i)(-2i-1)} = \dfrac{-1-2i+3i+6i^2}{-1-2i+2i+4i^2} = \dfrac{-1+i-6}{-1-4} = \dfrac{-7+i}{-5}$

   $= \dfrac{7}{5} - \dfrac{1}{5}i$

5. $\dfrac{i}{5+4i} \times \dfrac{4i-5}{4i-5} = \dfrac{i(4i-5)}{(5+4i)(4i-5)} = \dfrac{-5i+4i^2}{-25+20i-20i+16i^2} = \dfrac{-5i-4}{-25-16} = \dfrac{-5i-4}{-41}$

   $= \dfrac{4}{41} + \dfrac{5}{41}i.$