Operations

Addition:

Matrices can only be added if they are the same size. We add them by adding up numbers that are in the same position, for example:

$$\begin{pmatrix} 1 & 0 & 3 \\ 2 & 9 & 4 \end{pmatrix} + \begin{pmatrix} 6 & 5 & 8 \\ 0 & 1 & 7 \end{pmatrix} = \begin{pmatrix} 1+6 & 0+5 & 3+8 \\ 2+0 & 9+1 & 4+7 \end{pmatrix} = \begin{pmatrix} 7 & 5 & 11 \\ 2 & 10 & 11 \end{pmatrix}$$

Subtraction is performed similarly, for example:

$$\begin{pmatrix} 4 & 6 \\ 7 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 4-1 & 6-3 \\ 7-2 & 1-5 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 5 & -4 \end{pmatrix}$$

Scalar multiplication:

When we multiply a matrix by a scalar (a number or value that is not a matrix, for example 2 or a) we simply multiply each entry in the matrix by the scalar. For example:

$$p \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} pa & pb \\ pc & pd \end{pmatrix}$$

Multiplication:

Matrices can only be multiplied if the number of columns in the left matrix is equal to the number of rows in the right matrix.

The resulting matrix will have the same number of rows as the left matrix and the number of columns of the right matrix.

To multiply two matrices, we multiply the entries in the rows of the left matrix by the columns of the right matrix, as follows:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & cf+bh \\ ce+dg & cf+dh \end{pmatrix}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \begin{pmatrix} g & h \\ i & j \\ k & l \end{pmatrix} = \begin{pmatrix} ag+bi+ck & ah+bj+cl \\ dg+ei+fk & dh+ej+fl \end{pmatrix}$$

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Transpose:

Denoted A^T, the transpose of a matrix is formed by turning the rows of the matrix into columns and the columns into rows. For example:

$$\begin{pmatrix} 1 & 3 & 2 \\ 4 & 2 & 5 \end{pmatrix}^T = \begin{pmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{pmatrix}$$

Transpose properties:

$(A^T)^T = A$

$(A + B)^T = A^T + B^T$

$(AB)^T = B^T A^T$

$(cA)^T = cA^T$ (for a scalar c)

$(A^T)^{-1} = (A^{-1})^T$ (where A^{-1} is the inverse of A)

Trace:

The trace of a square matrix, $\text{tr}(A)$ is the sum of the entries on the diagonal. For example:

$$\text{tr}\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix} = 3 + 2 = 5$$

Trace properties:

$\text{tr}(A + B) = \text{tr}(A) + \text{tr}(B)$

$\text{tr}(AB) = \text{tr}(BA)$

$\text{tr}(cA) = c\,\text{tr}(A)$ (where c is a scalar)

$\text{tr}(A^T) = \text{tr}(A)$

$\text{tr}(A^T B) = \text{tr}(AB^T) = \text{tr}(B^T A) = \text{tr}(BA^T)$

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Inverse of a matrix:

We can only find an inverse of a square matrix.

The inverse of a matrix A is denoted A^{-1} and holds the special property that $AA^{-1} = A^{-1}A = I_n$.

For smaller matrices, we can use the $AA^{-1} = I_n$ property, and then solve for A^{-1}. For example:

To find the inverse of the matrix $\begin{pmatrix} 1 & 0 \\ 4 & 3 \end{pmatrix}$, we substitute in $A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and solve for a, b, c, d:

$$AA^{-1} = \begin{pmatrix} 1 & 0 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

So, we have $\begin{pmatrix} a & b \\ 4a + 3c & 4b + 3d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, which we solve to get

$a = 1, \qquad b = 0$

$4(1) + 3c = 0$, and so $c = \frac{-4}{3}$

$4(0) + 3d = 1$, and so $d = \frac{1}{3}$.

Therefore, $A^{-1} = \begin{pmatrix} 1 & 0 \\ -\frac{4}{3} & \frac{1}{3} \end{pmatrix}$, or $A^{-1} = \frac{1}{3}\begin{pmatrix} 3 & 0 \\ -4 & 1 \end{pmatrix}$.

Determinant:

We can only find a determinant for a square matrix.

The determinant of a matrix is a number that tells us certain properties of the matrix.

To calculate the determinant of a 2x2 matrix, we find:

$$det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

For a 3x3, we find:

$$det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a\left(det\begin{pmatrix} e & f \\ h & i \end{pmatrix}\right) - d\left(det\begin{pmatrix} b & c \\ h & i \end{pmatrix}\right) + g\left(det\begin{pmatrix} b & c \\ e & f \end{pmatrix}\right)$$

Determinant properties:

A matrix is invertible if and only if its determinant is not zero.

If the determinant of a matrix is zero, this means that the rows or columns are not linearly independent.

$det(I) = 1$

$det(cA) = c^n \, det(A)$ (for an $n \times n$ matrix)

$det(A^T) = det(A)$

$det(AB) = det(A) \, det(B)$

$det(A^{-1}) = \dfrac{1}{det(A)}$

If we swap two columns with each other in the matrix, we multiply the determinant by -1.

Adding a scalar multiple of one row to another row does not change the value of the determinant.

If there is a row or column of the matrix that is all zeros, then the determinant is 0.

Transformations

Stretch/compression:

A stretch/compression in the x direction by a factor of k is represented by a matrix $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$.

A stretch/compression in the y direction by a factor of k is represented by a matrix $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$.

Rotation:

A rotation of a position vector θ radians or degrees clockwise about the origin is represented by a

matrix $\begin{pmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{pmatrix}$

Reflection

A reflection of a position vector in the line $y = tan(\theta)$ is given by a matrix $\begin{pmatrix} cos(2\theta) & sin(2\theta) \\ sin(2\theta) & -cos(2\theta) \end{pmatrix}$

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