YORK ST JOHN UNIVERSITY
Est. 1841

Student Life

Library and Learning Services

Vectors

Study Development Worksheet

Questions

1. Calculate:

a. $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$

b. $\begin{pmatrix} 1 \\ 0 \\ -2 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \\ 1 \\ -3 \end{pmatrix}$

c. $\begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

d. $\begin{pmatrix} 13 \\ 12 \\ 27 \\ 2 \end{pmatrix} - \begin{pmatrix} 10 \\ -1 \\ 30 \\ -10 \end{pmatrix}$

e. $\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$

f. $(1 \quad 3 \quad -2) + (-3 \quad 10 \quad 4)$

g. $\begin{pmatrix} -1 \\ 10 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 2 \\ 11 \end{pmatrix}$

h. $\begin{pmatrix} \frac{1}{2} \\ -1 \\ \frac{3}{2} \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$

i. $3\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

j. $\frac{5}{4}\begin{pmatrix} 8 \\ -2 \end{pmatrix}$

k. $-\begin{pmatrix} 2 \\ 10 \\ -3 \\ 2 \end{pmatrix} + 3\begin{pmatrix} -1 \\ 3 \\ 12 \\ 1 \end{pmatrix}$

l. $\left\| \begin{pmatrix} 3 \\ 2 \\ -1 \\ 2 \end{pmatrix} \right\|$

m. $\left\| \begin{pmatrix} -4 \\ 2 \\ 1 \end{pmatrix} \right\|$

n. $\left\| \begin{pmatrix} 5 \\ 10 \end{pmatrix} \right\| + \left\| \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right\|$

o. $\begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} \times \begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix}$

p. $\begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix} \times \begin{pmatrix} -1 \\ 3 \\ 12 \end{pmatrix}$

q. $< \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix} >$

r. $< \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} >$

2. Calculate the angle (in radians) between:

   a. $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

   b. $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}$

   c. $\begin{pmatrix} 10 \\ 3 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -6 \\ 20 \\ 7 \end{pmatrix}$

   d. $\begin{pmatrix} 3 \\ 10 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

   e. $\frac{1}{3}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and $\frac{1}{\sqrt{73}}\begin{pmatrix} -1 \\ 6 \\ -6 \end{pmatrix}$

   f. $\begin{pmatrix} 7 \\ 1 \\ 0 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 3 \\ 4 \\ -1 \end{pmatrix}$

   g. Which of these pairs of vectors are orthogonal?

   h. Which are orthonormal?

3. Find the projection of:

   a. $\begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ onto $\begin{pmatrix} -1 \\ 7 \\ 3 \end{pmatrix}$

   b. $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ onto $\begin{pmatrix} 2 \\ 7 \end{pmatrix}$

   c. $\begin{pmatrix} 0 \\ 1 \\ 3 \\ -2 \end{pmatrix}$ onto $\begin{pmatrix} 1 \\ 3 \\ -1 \\ 6 \end{pmatrix}$

4. Find

   a. The length of the vector $(2 \quad -1 \quad 7 \quad 1)$

   b. A vector that is perpendicular to the vectors $\begin{pmatrix} 10 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ 3 \\ 1 \end{pmatrix}$.

Library and Learning Services

Study Development

Email: studydevelopment@yorksj.ac.uk

YORK
Est. 1841
ST JOHN
UNIVERSITY

Answers

1. Calculate:

   a. $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 2+1 \\ 1+0 \\ 1+3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$

   b. $\begin{pmatrix} 0 \\ 3 \\ -1 \\ 1 \end{pmatrix}$

   c. $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

   d. $\begin{pmatrix} 3 \\ 13 \\ -3 \\ 12 \end{pmatrix}$

   e. $\begin{pmatrix} 7 \\ 0 \\ 5 \end{pmatrix}$

   f. $(-2 \quad 13 \quad 2)$

   g. $\begin{pmatrix} -2 \\ 25 \end{pmatrix}$

   h. $\begin{pmatrix} 2 \\ -\frac{3}{2} \\ \frac{7}{2} \end{pmatrix}$

   i. $\begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$

   j. $\begin{pmatrix} 10 \\ \frac{-5}{2} \end{pmatrix}$

   k. $\begin{pmatrix} -5 \\ -1 \\ 39 \\ 1 \end{pmatrix}$

l. $3\sqrt{2}$

m. $\sqrt{21}$

n. $5\sqrt{5} + \sqrt{10}$

o. $\begin{pmatrix} -12 \\ 4 \\ -7 \end{pmatrix}$

p. $\begin{pmatrix} -33 \\ -17 \\ 5 \end{pmatrix}$

q. 7

r. 3

2. Calculate the angle (in radians) between:

a. $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\theta = cos^{-1}\left(\frac{<\binom{1}{3},\binom{3}{-1}>}{|\binom{1}{3}||\binom{3}{-1}|}\right) = cos^{-1}\left(\frac{0}{10}\right) = 0$

b. $cos^{-1}\left(\frac{-17}{2\sqrt{91}}\right) = 2.670^c$

c. 0

d. 0.755^c

e. 0

f. 0

g. a, c, e, f

h. e

3. Find the projection of:

a. $\text{proj}_{(-1\ \ 7\ \ 3)}\left(\begin{pmatrix}1\\1\\4\end{pmatrix}\right) = \frac{\begin{pmatrix}1\\1\\4\end{pmatrix}\cdot\begin{pmatrix}-1\\7\\3\end{pmatrix}}{\begin{pmatrix}-1\\7\\3\end{pmatrix}\cdot\begin{pmatrix}-1\\7\\3\end{pmatrix}}\begin{pmatrix}-1\\7\\3\end{pmatrix} = \frac{18}{21}\begin{pmatrix}-1\\7\\3\end{pmatrix} = \frac{6}{7}\begin{pmatrix}-1\\7\\3\end{pmatrix}$

b. $\text{proj}_{(2\ \ 7)}\left(\begin{pmatrix}3\\-1\end{pmatrix}\right) = \frac{-1}{18}\begin{pmatrix}2\\7\end{pmatrix}$

c. $\text{proj}_{(1\ \ 3\ \ -1\ \ 6)}\left(\begin{pmatrix}0\\1\\3\\-2\end{pmatrix}\right) = \frac{-12}{47}\begin{pmatrix}1\\3\\-1\\6\end{pmatrix}$

4. Find

a. $\left\|(2\ \ -1\ \ 7\ \ 1)\right\| = \sqrt{2^2 + (-1)^2 + 7^2 + 1^2} = \sqrt{4 + 1 + 49 + 1} = \sqrt{55}$

b. A perpendicular vector is given by the cross product of the vectors.

$$\begin{pmatrix}10\\0\\1\end{pmatrix} \times \begin{pmatrix}7\\3\\1\end{pmatrix} = \begin{pmatrix}0\times 1 - 3\times 1\\7\times 1 - 10\times 1\\3\times 10 - 0\times 7\end{pmatrix} = \begin{pmatrix}-3\\-3\\30\end{pmatrix}$$

Support: Study Development offers workshops, short courses, 1 to 1 and small group tutorials.

- Join a tutorial or workshop on the Study Development tutorial and workshop webpage or search 'YSJ study development tutorials.'

- Access our Study Success resources on the Study Success webpage or search 'YSJ study success.'

Library and Learning Services

Study Development

Email: studydevelopment@yorksj.ac.uk

YORK
ST JOHN
UNIVERSITY
Est. 1841