Right-angle trigonometry

Study Development Factsheet

Identifying triangles

There are many different types of triangle. We will look at a few that you should be able to recognise:

- Equilateral triangle. This is a triangle whose sides are all the same length, and internal angles are all 60° (or $\frac{\pi}{3}$ radians).
- Isosceles triangle. A triangle which has two sides that are the same length, and two internal angles which are the same.
- Scalene triangle. A triangle whose side lengths and internal angles are all different.
- Right-angle triangle. A triangle whose side lengths follow Pythagoras's Theorem and one of the internal angles is 90° (or $\frac{\pi}{2}$ radians).

A triangle could come under two different categories, for example, a right-angle triangle could be isosceles or scalene.

Triangle vocabulary

- Internal angle: the angle between two sides, measured on the inside of the triangle.
- External angle: the angle between two sides, measured on the outside of the triangle.
- Hypotenuse: the longest side/ the side that is opposite the right angle. This term is only used for right angle triangles.
- Opposite side: the side that is opposite whichever angle we are interested in. When referring to right-angle triangles, we do not label the hypotenuse as an 'opposite' side.
- Adjacent side: one of the sides that meet to make the angle we are interested in. When referring to right-angle triangles, we do not label the hypotenuse as an 'adjacent' side.

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Trigonometric ratios (SOHCAHTOA)

We can also deduce information about the angles in a right-angle triangle from their side lengths, and vice versa. We do this using the three trigonometric ratios:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

The hypotenuse is the longest side length (which will be the one opposite the right angle), the opposite side is the one that sits opposite the angle θ, and the adjacent side is the side length that is used along with the hypotenuse to create the angle θ.

sin is short for sine, cos is short for cosine and tan is short for tangent. They are three functions that we will likely need to use many times in many different ways.

We call them the SOHCAHTAO (pronounced so ca toe a) to try to remember the ratios. The ‘O’s are opposite sides, the ‘A’s are adjacent sides, and the ‘H’s are hypotenuses.

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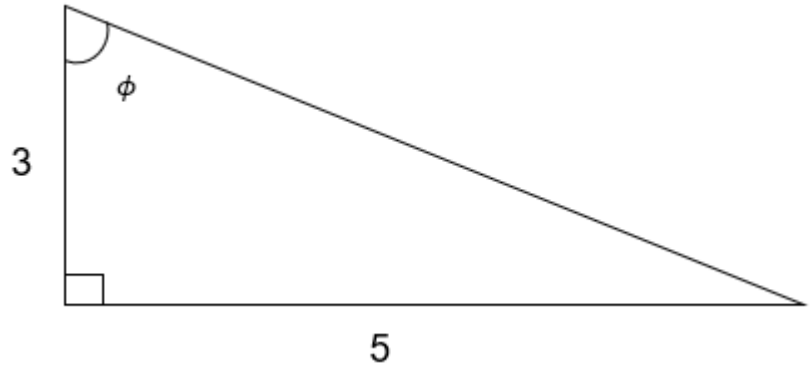
Example 1

A problem that we could use one of these to solve could be, for example:



Find the size of the angle ϕ.

First, we need to identify the types of side lengths we have. Here, the side length 3 is adjacent to the angle, and the side length 5 is opposite the angle.

Now we choose the trigonometric ratio that combines those two, in this case, $\tan(\phi) = \frac{\text{opposite}}{\text{adjacent}}$

Next, we fill in the side lengths: $\tan(\phi) = \frac{5}{3}$.

Each of these trigonometric functions has an 'inverse' (ie a function that you can apply to them to give you a value for θ). These are $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$, and are read as either 'sine to the minus one' or 'arc sine'. You can find these on a calculator (usually by pressing the shift key, and then the sin, cos or tan button).

So, we use the inverse of the tan function:

$$\tan^{-1}(\tan(\phi)) = \tan^{-1}\left(\frac{5}{3}\right)$$

$$\phi = \tan^{-1}\left(\frac{5}{3}\right) = 59.04°$$

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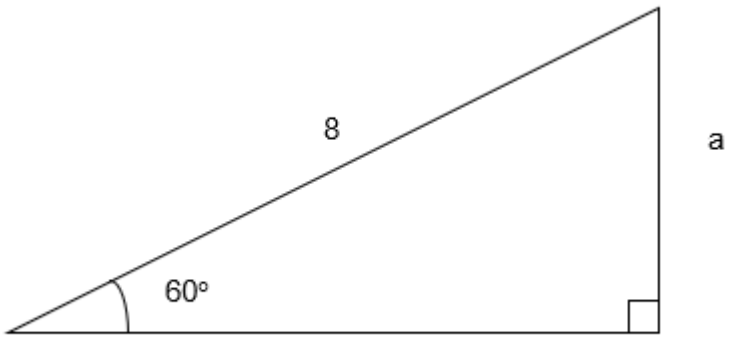
Example 2

We can also find a side length given another side length and an angle. For example:



The side of length 8 is the hypotenuse, and the side length a is the opposite side to the angle given, so we now select the ratio that combines the two:

$$\sin(60) = \frac{a}{8}$$

We put sin(60) into a calculator, to get $\sin(60) = \frac{1}{2}$. Therefore, we have:

$$\frac{1}{2} = \frac{a}{8}$$

So,

$$8 \times \frac{1}{2} = a = 4$$

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