Arithmetic and geometric sequence and series

Study Development Worksheet

## Questions

1. The first three terms in an arithmetic sequence are 1, 7 and 13. Find the common difference.
2. The second, third, and fourth terms in an arithmetic sequence as 6, 8 and 10. What is the first term and the common difference?
3. In an arithmetic sequence, the first three terms are , and . If , write an expression for
4. The first three terms in a geometric sequence are 3, 6 and 12. What is the common ratio of the sequence?
5. The third, fourth, and fifth terms in a geometric sequence are 2, 4 and 8. What is the first term and the common difference?
6. The first three terms of a geometric sequence are and . Given that , and , find a suitable value for and the common ratio .
7. A ball is bounced on the floor. After the first bounce, the ball reaches a maximum height of 2m. After each bounce, the height that the ball reaches decreases by 30%.
8. What height does the ball reach after 6 bounces?
9. At what point does the height of the ball go below 70cm?
10. What is the sum to infinity of the height of the bounces?

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## Answers

1. The general formula for the terms in an arithmetic sequence is where is the common difference. We begin by writing out the terms in this format:

Therefore, using , we can see that

1. We write out the terms using the general formula:

We find an expression for by finding which gives us . We then use to find which gives us

Therefore, an expression for is given by

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1. We write out the terms using the general formula:

We then find two expressions for in terms of

We then equate the two expressions for

Which we rearrange to get a quadratic:

Since there is a common factor of 5 in each term, we divide through by 5:

Which we factorise to get

Which gives us two possible values of : and

When we use to find , we get , which contradicts the question that states that , so we know that since this gives us

Therefore, , so the expression for is given by

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1. The general formula for a term in a geometric sequence is We write out each term using this formula:

We then find . Therefore, an expression for

1. We write out the terms using the general formula:

We then find an expression for by finding

We use this to find a value for by which gives

Therefore, we can find an expression for :

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1. We write out the terms using the general formula:

We then find two expressions for in terms of

We then equate the two expressions:

We cross multiply to get:

Then divide both sides by

We expand and rearrange:

Then we factorise to get:

So, our potential values for are

By the conditions in the question, we need to be greater than 0, which means cannot be or

We also need to give us . Since , we know that

Therefore,

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We then find

1. This question is eluding to us writing a geometric sequence formula for the height of the bounce. Since the height of the bounce is decreasing by 30% each time, we know that each bounce height is 70% of the bounce height before it. So, each bounce height is 0.7 times the height of the bounce before it. This is, therefore, our common ratio. We have our first term, as the first bounce is 2m high. We use the general formula to give an expression for the height after bounces:
2. We find how many bounces the ball has gone through when the height is exactly 0.7m:

Which we rearrange to get:

We use a logarithm function to find:

Since we can’t have 2.943 bounces, we round to the next highest whole number, which is 3.

1. Since , we can use this formula to find the sum to infinity of the heights:

Note: If you have not seen the logarithm function before, there are some worksheets on it in the maths success section.

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