Simultaneous Equations

Study Development Quick Guide

Solving simultaneous equations

Direct substitution

If you have an equation that gives a value for one variable (or can be rearranged to give you that) you can directly sub it into the other equation. For example:

$$x = 4y + 2$$

$$\frac{x}{2} + 1 = y + 3$$

Since we have an expression for x, we can just sub it directly into the second equation:

$$\frac{4y + 2}{2} + 1 = y + 3$$

Which becomes:

$$2y + 1 + 1 = y + 3$$

Which rearranges to give:

$$y = 1$$

and so

$$x = 4(1) + 2 = 6$$

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Addition and subtraction

We decide how to do this by looking at the coefficients of the variables.

For example:

$$3s - 2 = 8t$$

$$s + 1 = 6t$$

We minus $3 \times$ the second equation from the first, to get:

$$3s - 2 - 3(s + 1) = 8t - 3(6t)$$

Which becomes:

$$3s - 3s - 2 - 3 = 8t - 18t$$

Which simplifies to:

$$-5 = -10t$$

and so

$$t = \frac{1}{2}$$

Which gives us

$$3s - 2 = 8\left(\frac{1}{2}\right)$$

$$3s = 6$$

$$s = 2$$

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Matrix method

For the simultaneous equations

$$3p + 2 = 2q - 1$$

$$p + 7 = 4q$$

We rearrange to have the constants on one side and the variables on the other:

$$3p - 2q = -3$$

$$p - 4q = -7$$

We then set up a system of matrices in this format:

$$\begin{pmatrix} 3 & -2 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -3 \\ -7 \end{pmatrix}$$

We then find an inverse for the 2x2 matrix- there are many ways to do this, see the factsheet on matrices for more information.

The inverse of $\begin{pmatrix} 3 & -2 \\ 1 & -4 \end{pmatrix}$ is $\begin{pmatrix} \frac{4}{10} & \frac{-2}{10} \\ \frac{1}{10} & \frac{-3}{10} \end{pmatrix}$. We left-multiply both sides of the equation to get:

$$\begin{pmatrix} \frac{4}{10} & \frac{-2}{10} \\ \frac{1}{10} & \frac{-3}{10} \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{4}{10} & \frac{-2}{10} \\ \frac{1}{10} & \frac{-3}{10} \end{pmatrix} \begin{pmatrix} -3 \\ -7 \end{pmatrix}$$

Which simplifies to:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \frac{1}{5} \\ \frac{9}{5} \end{pmatrix}$$

Which gives us

$$p = \frac{1}{5}$$

$$q = \frac{9}{5}$$

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'Wordy' problems (real-world questions)

In order to solve a 'wordy' problem, try the following steps:

1. Decide what the variables should be.
2. Set up your two equations using the information given.
3. Solve using your preferred method.
4. Remember, when writing your answers, you might need to put units in.

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