Simultaneous equations

Study Development Worksheet

Questions

1. Find the coordinates of the point where the equations

$$3x + 4 = y$$
$$7x = y$$

intersect.

2. Find the coordinates of the point where the equations

$$3x + 2 = 4y$$
$$x - 1 = y$$

intersect.

3. Alex is 12 years older than his brother Brian. The sum of their ages is 72. How old is Alex?

4. You ask your friend to go to the shop and buy you some bags of your favourite sweets and some packets of your favourite crisps. You give your friend £5. She texts you when she's at the shop to say you can either have 2 bags of sweets and 2 packets of crisps, or 1 bag of sweets and 6 packets of crisps. Both combinations cost exactly £5 each. How much does 1 bag of sweets cost?

5. Find the coordinates of the points where the equations

$$xy = 40$$
$$x - 3 = y$$

meet.

6. A rectangle has two shorter sides of length $3y - 1$ and x, and two longer sides of length $4y + 2$ and $3x$. What is the area of the rectangle?

Simultaneous equations

7. Find the coordinates of the point where the equations

$$x + y + 2z = 8$$

$$x + z = 0$$

$$-x + 3y - z = 21$$

   meet.

8. Find the coordinates of the point where the equations

$$x + 4y + z = 10$$

$$3x - y - 2z = -5$$

$$-2x + 8y + z = 1$$

   meet.

Answers

1. We label the equations: (1) $3x + 4 = y$, and (2) $7x = y$.

   Then find (2) - (1): $7x - 3x - 4 = y - y$, and then simplify this to get $4x - 4 = 0$ which implies that $x = 1$.

   We then use (2) to get that $7(1) = y$, so $y = 7$.

   Therefore, the point at which the functions intersect is $(1,7)$.

2. Label the equations: (1) $3x + 2 = 4y$ and (2) $x - 1 = y$.

   Find (1) – 3 × (2): $3x + 2 - 3x + 3 = 4y - 3y$, and then simplify this to get $5 = y$.

   We then use (2) to give us $x - 1 = 5$, which implies that $x = 6$.

   Therefore, the point at which the functions intersect is $(6,5)$.

3. Say that Alex is a years old, and Brian is b years old. We can then construct the simultaneous equations:

   (1) $a = b + 12$

   (2) $a + b = 72$

   We calculate (2) – (1): $a + b - a = 72 - b - 12$, which we simplify to get $b = 60 - b$, and rearrange to get $2b = 60$, so $b = 30$.

   We then use (1) to find: $a = 30 + 12 = 42$.

   Therefore, Alex is 42 years old.

4. Say that the sweets cost £S and the crisps cost £C.

   We construct the simultaneous equations:

   (1) $2S + 2C = 5$

   (2) $S + 6C = 5$

   We find (1) – 2(2): $2S + 2C - 2S - 12C = 5 - 10$, which we simplify to get $-10C = -5$, which implies that $C = 0.5$.

   We then use (1) to find that $S + 3 = 5$, so we have that $S = 2$.

   So, we know that 1 bag of sweets costs £2.

5. (1) $xy = 40$

   and $(2)\ x - 3 = y$

   We substitute (2) into (1) to get $x(x-3) = 40$, which simplifies to $x^2 - 3x - 40 = 0$. We solve the quadratic to get $(x-8)(x+5) = 0$, so we have $x_1 = 8$, and $x_2 = -5$.

   Using (2) this gives us $8 - 3 = 5 = y_1$ and $-5 - 3 = -8 = y_2$.

   So, the points of intersection of the two functions are $(8,5)$ and $(-5,-8)$.

6. Since the two shorter side lengths will be equal in length, we get our first equation: (1) $3y - 1 = x$. The two longer side lengths will also be equal in length, so we get our second equation: (2) $4y + 2 = 3x$.

   We then solve these by finding (2) - 3(1): $4y + 2 - 9y + 3 = 3x - 3x$, which simplifies to give $-5y + 5 = 0$ which gives $y = 1$.

   We then use (1) to get $3 - 1 = 2 = x$.

   Therefore, the side lengths of the rectangle are 2 and 6, so the area is $2 \times 6 = 12$.

7. We begin by labelling the equations: (1) $x + y + 2z = 8$, (2) $x + z = 0$, (3) $-x + 3y - z = 21$.

   Using (2) we have that $x = -z$, so we replace x in equations (1) and (3) with $-z$, to get: (1) $-z + y + 2z = 8$, which means that (1) $y + z = 8$, and (2) $z + 3y - z = 21$, which gives (2) $3y = 21$, so $y = 7$.

   Then, using (1) we find $y + z = 7 + z = 8$, so $z = 1$. Finally, using (2) we find that $x = -z = -1$.

   Therefore, the point of intersection is $(-1,7,1)$.

8. We begin by labelling the equations: (1) $x + 4y + z = 10$, (2) $3x - y - 2z = -5$, (3) $-2x + 8y + z = 1$.

   We then eliminate z by finding (1) - (3): $x + 4y + z + 2x - 8y - z = 10 - 1$, which simplifies to $3x - 4y = 9$, which we label as (4).

   We now need another equation using just x and y, so we find (2) + (2 × (3)):
   $3x - y - 2z - 4x + 16y + 2z = -5 + 2$, which simplifies to $-x + 15y = -3$, and we label this as (5).

   We then want to use (4) and (5) to get an equation that only uses x or y, so we find (4) + (3 × (5)): $3x - 4y - 3x + 45y = 9 - 9$, which simplifies to $41y = 0$, so we have $y = 0$.

   We then use (5) to get $-x + 15(0) = -3$, which gives us $x = 3$.

   Finally, we use (3) to get $-2(3) + 8(0) + z = 1$, and so $z = 7$.

   Therefore, the point of intersection of the three functions is $(3,0,7)$.

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