Differentiation

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## Simple differentiation

Differentiate:

## Common differentiation rules

Differentiate:

## The chain rule

Differentiate:

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## The product rule

Differentiate:

## The quotient rule

Differentiate:



## Equation of a tangent

Find the tangent of the function at the point when:

1. and
2. and
3. and
4. and

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## Turning points

Find (and define) the turning points of the functions:

1. where

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## Answers

## Simple differentiation

## Common differentiation rules



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=

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## The chain rule

1. with .

, and .

Therefore, .

1. with .

, and

Therefore,

1. with .

, and

Therefore,

1. with

, and

Therefore,

1. with

, and

Therefore,

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## The product rule

1. and .

and .

Therefore, .

1. and

and

Therefore,

Of course, you could also do this by expanding the brackets to get and then differentiating that to get

1. and

and (you can find this using the chain rule).

Therefore,

1. and

and

Therefore,

1. and

and (you can find this using the chain rule).

Therefore,

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## The quotient rule

1. and

and

Therefore,

1. and

and

Therefore, .

1. and

and

Therefore, = .

1. and .

and

Therefore, = .

1. and

and

Therefore, = .

1. and

and

Therefore, we have that .

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1. .

and

and

Therefore, we have that = = .

## Equation of a tangent

So, the gradient at is equal to , and we have the equation for the tangent as

We find :

, so , and the equation of the tangent is

So, the gradient at is equal to and we have the equation for the tangent as

We find

, so, the equation of the tangent is

1. (you can find this by using the chain rule).

So, the gradient at is , and we have the equation for the tangent as

We find

so , and the equation of the tangent is

So, the gradient at is equal to and we have the equation for the tangent as

We find

, so , and the equation of the tangent is

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## Turning points

Find (and define) the turning points of the functions:

gives us , and so we have and

We find the corresponding values:

and .

So, the two turning points of are and

To define the turning points, we differentiate to get

We now find Since this is greater than zero, we know that is a minima. We find Since this is less than zero, we know that is a maxima.

gives us . We find the corresponding value: So, the turning point of is

To define the turning point, we differentiate to get Since this is greater than zero at every point, we know that the turning point is a minima.

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1. Call . We find using the quotient rule:

and , therefore and . So, we have that .

Since = , we have that = .

gives us that , so and Since it is stated that we discard . We find the corresponding value:

, so, we have that the turning point of the function exists at

To define the turning point, we differentiate to get = (using the quotient rule).

We find , so we have that the turning point is a maxima.

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gives us that . We also have that could be equal to , and so on. In fact, is equal to for any integer

We find the corresponding values:

, and in fact, for all even

and for all odd

So, the turning points of exist at and for all integer values of

To define these turning points, we differentiate to give

and for all integer values of

Therefore, we know that the turning points of the form are maxima and all turning points of the form are minima.

**Note:** If you found the last two questions (or indeed any of the questions in this worksheet) to be difficult, don’t worry! Differentiation takes a lot of practice, and this worksheet contains some quite tricky differentiation questions. If you are less confident with the later questions, try to focus on the simple differentiation section until you feel happy with that.

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