Logarithms

Study Development Worksheet

Example

Simplify the following:

$\ln(12) - \ln(10)$

Answer

Using the log laws, we know that

$$\ln(12) - \ln(10) = ln\left(\frac{12}{10}\right) = ln\left(\frac{6}{5}\right) = ln(1.2)$$

Questions

1. Evaluate:
   i) $log_4(16)$
   ii) $log_2(32)$
   iii) $log_3\left(\frac{1}{3}\right)$
   iv) $ln(1)$
   v) $ln(10)$
   vi) $log_1(5)$
   vii) $log_2(5)$
   viii) $log_9\left(\frac{1}{27}\right)$

2. Calculate y in each of the following:
   i) $4^y = 64$
   ii) $3^y = \frac{1}{3}$
   iii) $2^y = 256$
   iv) $1^y = 1$
   v) $5^y = 1$
   vi) $9^y = 0$
   vii) $27^y = 3$
   viii) $4^y = \frac{1}{8}$

3. Simplify:
   i) $log_a(15) + log_a(2)$
   ii) $ln(100) + ln(9) - ln(18)$
   iii) $log_2(1) + log_2(2)$
   iv) $ln(e^2) - ln(1) + 2\ln(e^2)$
   v) $4\,ln(2) - 2\,ln(4)$

4. Find a value for x in each of the following:
   i) $\log_x(25) = 2$
   ii) $x\,ln(e^3) = 9$
   iii) $log_x\left(\frac{1}{9}\right) = -2$
   iv) $ln(e^x) + \ln(e^2) = 6$
   v) $log_x(3) - log_x(27) = -1$

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Answers

1. Using log laws or a calculator, we find:
   i) $log_4(16) = 2$
   ii) $log_2(32) = 5$
   iii) $log_3\left(\frac{1}{3}\right) = log_3(1) - log_3(3) = 0 - 1 = -1.$

   Alternatively, you may already know that $3^{-1} = \frac{1}{3}$, in which case you do not need to separate the logs out.
   iv) $ln(1) = 0$
   v) $ln(10) = 2.303$
   vi) $log_1(5)$ does not exist. There exists no x such that $1^x = 5$.
   vii) $log_2(5) = 2.322$ (this is done using a calculator).
   viii) $log_9\left(\frac{1}{27}\right) = log_9(1) - log_9(27) = 0 - log_9(3^3) = -3\,log_9(3) = -3\left(\frac{1}{2}\right) = \frac{-3}{2}.$
2. By using the log function, we can calculate the answers:
   i) $4^y = 64$. Therefore, we calculate $y = log_4(64) = 3$.
   ii) $y = log_3\left(\frac{1}{3}\right) = -1.$
   iii) $y = log_2(256) = 8.$
   iv) $y = log_1(1) = 1$ (or any value).
   v) $y = log_5(1) = 0$
   vi) $y = log_9(0)$. Since $log_9(0)$ does not exist, there is no possible y that solves this equation.
   vii) $y = log_{27}(3) = \frac{1}{3}.$
   viii) $y = log_4\left(\frac{1}{8}\right) = log_4(1) - log_4(8) = 0 - \frac{3}{2} = \frac{-3}{2}.$

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3. Using log laws, we simplify the expressions.
   i) $log_a(15) + log_a(2) = log_a(15 \times 2) = log_a(30)$.
   ii) $ln(100) + ln(9) - ln(18) = ln(100 \times 9) - ln(18) =$
   $ln(900) - ln(18) = ln\left(\frac{900}{18}\right) = ln(50)$ (= 3.912).
   iii) $log_2(1) + log_2(2) = 0 + 1 = 1$.
   iv) $ln(e^2) - ln(1) + 2\,ln(e^2) = 2 - 0 + ln((e^2)^2) =$
   $2 + ln(e^4) = 2 + 4 = 6$.
   v) $4\,ln(2) - 2\,ln(4) = ln(2^4) - ln(4^2) = ln(16) - ln(16) = ln\left(\frac{16}{16}\right) = ln(1) = 0$.

4. We rearrange the equations and then find a value for x:
   i) $log_x(25) = 2$ rearranged gives: $x^2 = 25$, which means that $x = \sqrt{25} = \pm 5$.
   ii) $x\,ln(e^3) = 9$ simplified gives $3x = 9$, which means that $x = \frac{9}{3} = 3$.
   iii) $log_x\left(\frac{1}{9}\right) = -2$ is the same as $x^{-2} = \frac{1}{9}$, and so $\frac{1}{x^2} = \frac{1}{9}$. This gives us $x^2 = 9$, and so $x =$
   ± 3.
   iv) $\ln(e^x) + \ln(e^2) = x + 2 = 6$, and so $x = 6 - 2 = 4$.
   v) $log_x(3) - log_x(27) = log_x\left(\frac{3}{27}\right) = log_x\left(\frac{1}{3}\right) = -1$, which gives us $x^{-1} = \frac{1}{x} = \frac{1}{3}$, and
   therefore, $x = 3$.

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