# Complex numbers

Study Development Quick Guide

## Complex number arithmetic

There are several rules for manipulating complex numbers.

Addition:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

Subtraction:

$$(a + bi) - (c + di) = (a + c) + (b - d)i$$

Multiplication:

$$(a + bi) \times (c + di) = ac - bd + (ad + bc)i$$

Division:

$$\frac{a + bi}{c + di} = \frac{-ac - bd}{-c^2 - d^2} + \frac{(ad - bc)i}{-c^2 - d^2}$$

## Polar form

### Modulus

The 'modulus', $r$, is the length of the line between $z$ and the origin. We calculate this using Pythagoras's Theorem: $(|z|)^2 = x^2 + y^2$, so therefore $|z| = r = \sqrt{x^2 + y^2}$.

### Argument and quadrant adjustments

The 'argument' $arg(z)$ is the angle θ between z and the real axis. We calculate this using the tangent function: $tan(\alpha) = \frac{y}{x}$, and so $\alpha = tan^{-1}\left(\frac{y}{x}\right)$. We then adjust α based on the quadrant that $z$ is in to find the argument θ.

| Quadrant | x and y values | θ from α |
|---|---|---|
| 1st | x > 0, y > 0 | $\theta = \alpha$ |
| 2nd | x < 0, y > 0 | $\theta = \pi - \alpha$ |
| 3rd | x < 0, y < 0 | $\theta = \alpha - \pi$ |
| 4th | x > 0, y < 0 | $\theta = -\alpha$ |

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## Converting from polar form to Cartesian

If we are given a complex number in the form $z = r(cos(\theta) + i\,sin(\theta))$ or $z = re^{i\theta}$ and we want to put it into the form $z = x + yi$ we can follow these steps:

1. Calculate $x = r\,cos(\theta)$.
2. Calculate $y = r\,sin(\theta)$.
3. Write the number in the form $z = x + yi$.

## Converting from Cartesian to polar form

If we are given a complex number of the form $z = x + yi$, and we would like it in the form $z = r(cos(\theta) + i\,sin(\theta))$ or $z = re^{i\theta}$, we do the following:

1. Calculate $r = \sqrt{x^2 + y^2}$.
2. Calculate $\alpha = tan^{-1}\left(\frac{y}{x}\right)$.
3. Adjust $\alpha$ based on the quadrant of $z$ to get $\theta$.
4. Write the number in the form $z = r(cos(\theta) + i\,sin(\theta))$ or $z = re^{i\theta}$.

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