Rationalising the denominator and making the denominator real

Study Development Factsheet

## Rationalising the denominator

An irrational number is (informally) a number that we cannot write as a fraction with an integer numerator and denominator. For example, $\sqrt{2}$ is irrational.

For a number in the form ‘$a + y\sqrt{b}$’ where $\sqrt{b}$ is irrational, we call $'a'$ the ‘rational part’ and ‘$y\sqrt{b}$’ the ‘irrational part’.

In order to rationalise the denominator of a fraction that is in the form $'a + y\sqrt{b}'$, we use the following steps:

1. Take the irrational part of the denominator, and subtract the rational part. Call this value ‘c’.

Eg) For the fraction $\frac{1}{2 + \sqrt{7}}$ we would get $c = \sqrt{7} - 2$.

1. Multiply the fraction by $\frac{c}{c}$.

Eg)$\frac{1}{2 + \sqrt{7}} =$ $\frac{1}{2 + \sqrt{7}}×\frac{\sqrt{7} - 2}{\sqrt{7} - 2}$ = $\frac{1 \left(\sqrt{7} - 2\right)}{\left(2 + \sqrt{7}\right)\left(\sqrt{7} - 2\right)}$ =$ \frac{\sqrt{7} - 2}{2\sqrt{7} - 2^{2} + \left(\sqrt{7}\right)^{2}- 2\sqrt{7}}$.

**Note:** We are allowed to do this because $\frac{c}{c}$ = 1, and multiplying anything by 1 does not change its value.

1. Tidy up the numerator and denominator.

Eg) $\frac{\sqrt{7} - 2}{2\sqrt{7} - 2^{2} + \left(\sqrt{7}\right)^{2}- 2\sqrt{7}}$ = $\frac{\sqrt{7} - 2}{-4 + 7}$ = $\frac{\sqrt{7} - 2}{3}$.

1. Simplify the fraction, if you can.

Eg) For a fraction such as $\frac{6 + 3\sqrt{2}}{3}$ we would simplify to $2 + \sqrt{2}$.

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1. Change the format, if this is needed.

Eg) If we were asked to write $\frac{1}{2 + \sqrt{7}}$ in the form $a + y\sqrt{b}$, we would rationalise the denominator to give us $\frac{\sqrt{7} - 2}{3}$, and then write this as $\frac{-2}{3}$ + $\frac{\sqrt{7}}{3}$.

**Note**

$\sqrt{a} × \sqrt{a} = \left(\sqrt{a}\right)^{2}= a$.

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## Making the denominator real

This process is largely the same as rationalising the denominator.

An imaginary number is (informally) a number that has $\sqrt{-1}$ as a factor. We write $\sqrt{-1}$ as $i$.

A complex number is (informally) a number that has a real part and an imaginary part. For example, $3 + 2i$ is a complex number.

For a number in the form ‘$a + bi$’ where $i = \sqrt{-1}$, we call $'a'$ the ‘real part’ and ‘$bi'$ the ‘imaginary part’.

In order to make the denominator of a fraction that is in the form ‘$a + bi$’ real, we use the following steps:

1. Take the imaginary part of the denominator, and subtract the real part. Call this value ‘c’.

Eg) For the fraction $\frac{3}{1 + 2i}$ we would get $c = 2i - 1$.

1. Multiply the fraction by $\frac{c}{c}$.

Eg) $\frac{3}{1 + 2i}$ $×$ $\frac{2i - 1}{2i - 1}$ = $\frac{3\left(2i - 1\right)}{\left(1 + 2i\right)\left(2i - 1\right)}$ = $\frac{6i - 3}{2i - 1 + \left(2i\right)^{2} - 2i} $

**Note:** Again, we are allowed to do this because $\frac{c}{c}$ = 1, and multiplying anything by 1 does not change its value.

1. Tidy up the numerator and denominator.

Eg) $\frac{6i - 3}{2i - 1 + \left(2i\right)^{2} - 2i}$ = $\frac{6i - 3}{-1 + 4i^{2}}$ = $\frac{6i - 3}{-1 - 4}$ = $\frac{6i - 3}{-5}$.

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1. Simplify the fraction, if you can.

Eg) $\frac{6i - 3}{-5}$ = $\frac{3 - 6i}{5}$.

1. Change the format, if this is needed.

Eg) If we were asked to write $\frac{3}{1 + 2i}$ in the form $a + bi$, we would make the denominator real to give us $\frac{3 - 6i}{5}$, and then write this as $\frac{3}{5}$ - $\frac{6i}{5}$.

**Note**

Since $i=\sqrt{-1}$, we need to remember that:

$$i=\sqrt{-1}$$

$$i^{2}=\left(\sqrt{-1}\right)^{2}=-1$$

$$i^{3}=\left(\sqrt{-1}\right)\left(\sqrt{-1}\right)^{2}=\left(\sqrt{-1}\right)\left(-1\right)=-i$$

$$i^{4}=\left(\sqrt{-1}\right)^{2}\left(\sqrt{-1}\right)^{2}=\left(-1\right)\left(-1\right)=1$$

$$i^{5}=i^{4}×i=1×i=i$$

Which, without the calculation steps is:

$$i = i$$

$$i^{2}=-1$$

$$i^{3}=-i$$

$$i^{4}=1$$

$$i^{5}=i$$

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