YORK ST JOHN UNIVERSITY
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Student Life
Library and Learning Services

Rationalising the denominator and making the denominator real

Study Development Factsheet

Rationalising the denominator

An irrational number is (informally) a number that we cannot write as a fraction with an integer numerator and denominator. For example, $\sqrt{2}$ is irrational.

For a number in the form ‘a + y$\sqrt{b}$’ where $\sqrt{b}$ is irrational, we call ′a' the ‘rational part’ and ‘y$\sqrt{b}$’ the ‘irrational part’.

In order to rationalise the denominator of a fraction that is in the form ′a + y$\sqrt{b}$′, we use the following steps:

1. Take the irrational part of the denominator, and subtract the rational part. Call this value ‘c’.

   Eg) For the fraction $\frac{1}{2+\sqrt{7}}$ we would get $c = \sqrt{7} - 2$.

2. Multiply the fraction by $\frac{c}{c}$.

   Eg) $\frac{1}{2+\sqrt{7}} = \frac{1}{2+\sqrt{7}} \times \frac{\sqrt{7}-2}{\sqrt{7}-2} = \frac{1\,(\sqrt{7}-2)}{(2+\sqrt{7})(\sqrt{7}-2)} = \frac{\sqrt{7}-2}{2\sqrt{7}-2^2+(\sqrt{7})^2-2\sqrt{7}}$.

   Note: We are allowed to do this because $\frac{c}{c} = 1$, and multiplying anything by 1 does not change its value.

3. Tidy up the numerator and denominator.

   Eg) $\frac{\sqrt{7}-2}{2\sqrt{7}-2^2+(\sqrt{7})^2-2\sqrt{7}} = \frac{\sqrt{7}-2}{-4+7} = \frac{\sqrt{7}-2}{3}$.

4. Simplify the fraction, if you can.

   Eg) For a fraction such as $\frac{6+3\sqrt{2}}{3}$ we would simplify to $2+\sqrt{2}$.

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5. Change the format, if this is needed.

   Eg) If we were asked to write $\frac{1}{2+\sqrt{7}}$ in the form $a + y\sqrt{b}$, we would rationalise the denominator to give us $\frac{\sqrt{7}-2}{3}$, and then write this as $\frac{-2}{3} + \frac{\sqrt{7}}{3}$.

Note

$\sqrt{a} \times \sqrt{a} = \left(\sqrt{a}\right)^2 = a$.

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Making the denominator real

This process is largely the same as rationalising the denominator.

An imaginary number is (informally) a number that has $\sqrt{-1}$ as a factor. We write $\sqrt{-1}$ as i.
A complex number is (informally) a number that has a real part and an imaginary part. For example, 3 + 2i is a complex number.

For a number in the form 'a + bi' where $i = \sqrt{-1}$, we call 'a' the 'real part' and 'bi' the 'imaginary part'.

In order to make the denominator of a fraction that is in the form 'a + bi' real, we use the following steps:

1. Take the imaginary part of the denominator, and subtract the real part. Call this value 'c'.

   Eg) For the fraction $\frac{3}{1+2i}$ we would get c = 2i - 1.

2. Multiply the fraction by $\frac{c}{c}$.

   Eg) $\frac{3}{1+2i} \times \frac{2i-1}{2i-1} = \frac{3(2i-1)}{(1+2i)(2i-1)} = \frac{6i-3}{2i-1+(2i)^2-2i}$

   Note: Again, we are allowed to do this because $\frac{c}{c} = 1$, and multiplying anything by 1 does not change its value.

3. Tidy up the numerator and denominator.

   Eg) $\frac{6i-3}{2i-1+(2i)^2-2i} = \frac{6i-3}{-1+4i^2} = \frac{6i-3}{-1-4} = \frac{6i-3}{-5}$.

4. Simplify the fraction, if you can.

   Eg) $\frac{6i - 3}{-5} = \frac{3 - 6i}{5}$.

5. Change the format, if this is needed.

   Eg) If we were asked to write $\frac{3}{1 + 2i}$ in the form a + bi, we would make the denominator real to give us $\frac{3 - 6i}{5}$, and then write this as $\frac{3}{5} - \frac{6i}{5}$.

Note

Since $i = \sqrt{-1}$, we need to remember that:

$$i = \sqrt{-1}$$

$$i^2 = \left(\sqrt{-1}\right)^2 = -1$$

$$i^3 = \left(\sqrt{-1}\right)\left(\sqrt{-1}\right)^2 = \left(\sqrt{-1}\right)(-1) = -i$$

$$i^4 = \left(\sqrt{-1}\right)^2\left(\sqrt{-1}\right)^2 = (-1)(-1) = 1$$

$$i^5 = i^4 \times i = 1 \times i = i$$

Which, without the calculation steps is:

$$i = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

$$i^5 = i$$

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