Rationalising the denominator and making it real

Study Development Worksheet

## Rationalising the denominator

### Example

Rationalise the denominator of $\frac{2}{\sqrt{2}}. $

### Answer

$$\frac{2}{\sqrt{2}} × \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\left(\sqrt{2}\right)^{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

### Questions

Rationalise the denominator of:

1. $\frac{1}{\sqrt{3}}$
2. $\frac{3}{2\sqrt{5}}$
3. $\frac{2}{2 - \sqrt{3}}$
4. $\frac{5}{\sqrt{7} + 4}$
5. $\frac{10}{4\sqrt{2} + 9}$

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### Answers

1. $\frac{1}{\sqrt{3}}×\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{\left(\sqrt{3}\right)^{2}}$

$=\frac{\sqrt{3}}{3}$.

1. $\frac{3}{2\sqrt{5}}×\frac{2\sqrt{5}}{2\sqrt{5}}=\frac{6\sqrt{5}}{\left(2\sqrt{5}\right)^{2}}=\frac{6\sqrt{5}}{4×5}=\frac{6\sqrt{5}}{20}$

$=\frac{3\sqrt{5}}{10}$.

1. $\frac{2}{2 - \sqrt{3}}×\frac{-\sqrt{3} - 2}{-\sqrt{3} - 2} = \frac{2\left(-\sqrt{3} -2\right)}{\left(2 - \sqrt{3}\right)\left( -\sqrt{3} - 2\right)} = \frac{- 2\sqrt{3} - 4}{-2\sqrt{3} - 4 + \left(-\sqrt{3}\right)^{2}+ 2\sqrt{3}} = \frac{-4 - 2\sqrt{3}}{-4 + 3} = \frac{-4 - 2\sqrt{3}}{-1}$

$= 4 + 2\sqrt{3}$.

1. $\frac{5}{\sqrt{7} + 4}×\frac{\sqrt{7 }- 4}{\sqrt{7} - 4}$ $=\frac{5\left(\sqrt{7} - 4\right)}{\left(\sqrt{7} + 4\right)\left(\sqrt{7} - 4\right)}$ = $\frac{5\sqrt{7} - 20}{\left(\sqrt{7}\right)^{2} + 4\sqrt{7} - 4\sqrt{7} - 16}$ = $\frac{5\sqrt{7} - 20}{7 - 16}$ = $\frac{5\sqrt{7} - 20}{-9}$

$$=\frac{20 - 5\sqrt{7}}{9}$$

1. $\frac{10}{4\sqrt{2} + 9}×\frac{4\sqrt{2 } - 9}{4\sqrt{2} - 9}=\frac{10\left(4\sqrt{2 }- 9\right)}{\left(4\sqrt{2} + 9\right)\left(4\sqrt{2} - 9\right)}=\frac{40\sqrt{2} - 90}{\left(4\sqrt{2}\right)^{2} + 36\sqrt{2} - 36\sqrt{2 }- 81}=\frac{40\sqrt{2} - 90}{32 - 81}$

$$=\frac{40\sqrt{2} - 90}{-49}$$

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## Making the denominator real

### Example

Write $\frac{10}{4+3i}$ in the form $a + bi$.

### Answer

$$\frac{10}{4 + 3i}×\frac{3i - 4}{ 3i - 4}=\frac{10\left(3i - 4\right)}{\left(4 + 3i\right)\left(3i - 4\right)}=\frac{30i - 40}{12i - 16 + 9i^{2} + 12i}=\frac{-40 + 30i}{-16 - 9}=\frac{-40 + 30i}{-25}$$

$=\frac{-8 + 6i}{-5}=\frac{8}{5}-\frac{6}{5}i$ .

**Remember:**

### Questions

Write the following in the form a + bi:

1. $\frac{1}{5i}$
2. $\frac{-5}{2i - 1}$
3. $\frac{-37}{3 + \frac{i}{2}}$
4. $\frac{1 - 3i}{1 - 2i}$
5. $\frac{i}{5 + 4i}$

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### Answers

1. $\frac{1}{5i}×\frac{5i}{5i}=\frac{5i}{\left(5i\right)^{2}}=\frac{5i}{25i^{2}}=\frac{5i}{-25}$

$=\frac{-i}{5}$.

1. $\frac{-5}{2i - 1}×\frac{2i + 1}{2i + 1}=\frac{-5\left(2i + 1\right)}{\left(2i - 1\right)\left(2i + 1\right)}=\frac{-5 - 10i}{\left(2i\right)^{2} - 2i + 2i - 1}=\frac{-5 - 10i}{4i^{2} - 1}=\frac{-5 - 10i}{-4 - 1}=\frac{-5 - 10i}{-5}$

$$=1+2i.$$

1. $\frac{-37}{3 + \frac{i}{2}}×\frac{\frac{i}{2} - 3}{\frac{i}{2} - 3}=\frac{-37\left(\frac{i}{2} - 3\right)}{\left(3 + \frac{i}{2}\right)\left(\frac{i}{2} - 3\right)}=\frac{-37\left(\frac{i}{2} - 3\right)}{\frac{3i}{2} - 9 + \frac{i^{2}}{4} - \frac{3i}{2}}=\frac{-37\left( \frac{i}{2} - 3\right)}{-9 - \frac{1}{4}}=\frac{-37\left( \frac{i}{2} - 3\right)}{\frac{-37}{4}}=4\left(-3 + \frac{i}{2}\right)$

$$= -12+2i$$

1. $\frac{1-3i}{1-2i}×\frac{-2i - 1}{-2i - 1}=\frac{\left(1-3i\right)\left(-2i - 1\right)}{\left(1-2i\right)\left(-2i - 1\right)}=\frac{-1 - 2i + 3i + 6i^{2}}{-1 - 2i + 2i + 4i^{2}}=\frac{-1 + i - 6}{-1 - 4}=\frac{-7+i}{-5}$

$$=\frac{7}{5}-\frac{1}{5}i$$

1. $\frac{i}{5+4i}×\frac{4i - 5}{4i - 5}=\frac{i\left(4i - 5\right)}{\left(5+4i\right)\left(4i - 5\right)}=\frac{-5i + 4i^{2}}{-25 + 20i - 20i + 16i^{2}}=\frac{-5i - 4}{-25 - 16}=\frac{-5i - 4}{-41}$

$$=\frac{4}{41}+\frac{5}{41}i.$$

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