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Student Life
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Eigenvectors, eigenvalues and diagonalisation

Study Development Quickguide

Eigenvectors and eigenvalues

The eigenvectors and eigenvalues of a matrix are the solutions to the equation

$$Ax = \lambda x$$

Where A is a square matrix, x is an eigenvector of A and λ is an eigenvalue of A.

We might look at this equation and see that $x = 0$ is an obvious solution, no matter what A and λ are. Unfortunately, one of the properties of an eigenvector is that it cannot be the zero vector, so we must look for other solutions.

Characteristic equation

The first step in finding the eigenvalues of a matrix is to find its characteristic equation:

$$det(A - \lambda I) = 0$$

(See Factsheet for an explanation of why we find this).

Then, solve $det(A - \lambda I) = 0$ to find values for λ (the eigenvalues).

Eigenvectors

We find the eigenvectors by solving the equation

$$Ax = \lambda x$$

For each of the values of λ we have just found.

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Diagonalisation

Now that we have the eigenvectors and eigenvalues for a matrix, we can use a really cool property of matrices, to find the diagonalistion of the matrix.

Matrices can be diagonalised as follows:

$$A = PDP^{-1}$$

Where A is the matrix we wish to diagonalise, D is a diagonal matrix found from the eigenvalues of A and P is a matrix formed from the eigenvectors of A.

For example, when $\mathrm{A} = \begin{pmatrix} -5 & -6 \\ 3 & 4 \end{pmatrix}$, we find that

$$D = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$$

A matrix with the eigenvalues on the diagonals and zeros elsewhere.

We find P by writing the eigenvectors of A as columns of P, in the same order as the eigenvalues they correspond to in D (so in this case, an eigenvector corresponding to $\lambda_1 = 1$ would go in the first column, and an eigenvector corresponding to $\lambda_2 = -2$ goes in the second column):

$$P = \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$$

We find P^{-1} using your preferred method for finding an inverse:

$$P^{-1} = \begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix}$$

So, we have found that

$$A = PDP^{-1}$$

$$\begin{pmatrix} -5 & -6 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix}$$

Now we have the cool bit:

If you were asked to calculate A^6, you could probably do this, but it would take a really long time, and it'd be very easy to make a mistake somewhere. However, if we look at A^6 as:

$$A^6 = (PDP^{-1})^6$$

we find that we can actually just calculate $A^6 = PD^6P^{-1}$.

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