Eigenvectors and Eigenvalues

Study Development Worksheet

## Questions

1. Find the characteristic equations of the following matrices:
	1. For this question (and all matrices above dimension 3) most people would use an online solver. This question is thrown in for people who want a challenge, but you are welcome to use it just to practice using an online eigenvector and eigenvalue finder.
2. Using your answers to question 1, find the eigenvalues of the matrices:
3. Find an eigenvector corresponding to each of the eigenvalues of the matrices:

## Answers

* 1. Find the characteristic equations of the following matrices:

 To find the characteristic equation of an -dimensional matrix , we find :

* + 1. =

You can leave it like this (since it’s already factorised, which will help with question 2c, or if you expand and simplify this you’ll get .

* + 1.

You can leave it in this format since it’s already factorised, which will help with 2e, or you can expand and simplify to get:

1. Using your answers to question 1, find the eigenvalues of the matrices:
	1. To find the eigenvalues, we solve the characteristic equation of the matrix. factorises to give , and so the solutions/eigenvalues are and
2. Find an eigenvector of the matrices:
	1. To find an eigenvector corresponding each of the eigenvalues, we solve for each .

Which gives

If we write and these as simultaneous equations, we get

For :

Which both simplify to give

So, an eigenvector corresponding to could be any scalar multiple of

For , we have

Which we rearrange to get

So, an eigenvector corresponding to could be any scalar multiple of

* 1.

Therefore, an eigenvector of is any scalar multiple of

* 1. We find each solution to :

We multiply this out to get:

For :

, which implies that

, which implies that

, so we choose , which gives

Therefore, an eigenvector of which corresponds to is any scalar multiple of .

For :

, which implies that

, which implies that

, which has already been solved by

This means that can take any real value, so an eigenvector of corresponding to is any scalar multiple of .

Finally, for :

, so

, which implies that

, which implies that , which further implies that since

So, an eigenvector of corresponding to is any scalar multiple of .

For :

, which rearranges to get

, which implies that , which further implies that

, which is solved by the above .

So, an eigenvector of corresponding to is any scalar multiple of .

For :

, which rearranges to

, which rearranges to , so , which implies that , so

, which rearranges to

So, an eigenvector of corresponding to is any scalar multiple of .

For :

, which rearranges to , so

, which rearranges to

, which rearranges to

So, an eigenvector of corresponding to is any scalar multiple of .

For :

, which implies that

, which rearranges to

, which (with ) rearranges to

, which is already solved by

So, an eigenvector of corresponding to is any scalar multiple of.

For :

, which rearranges to , so

, which rearranges to

, which rearranges to , so

, which implies that , which further implies that

This leaves , so an eigenvector of corresponding to is any scalar multiple of .

Finally, for :

, which rearranges to

, so , and

, which (with ) rearranges to

, which is already solved by

So an eigenvector of corresponding to is any scalar multiple of .

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