Finding a basis for

Study Development Quickguide

## What is a basis?

(Informally) a basis of is a set of vectors in that you can make any vector in out of by finding a linear combination of the basis vectors.

## Checking if a set of vectors is a basis

When we are given a set of vectors and asked ‘is this set of vectors a basis in ?’ we are being asked to check two things:

* Are the vectors linearly independent?
* Do they span ?

If the answer to both is yes, then we have a basis for .

## Linear independence

A set of vectors being linearly independent means that we cannot form one of the vectors as a linear combination of the others. A linear combination of a set of vectors  is where are scalars.

For example,

, and are not linearly independent, since = .

We can test for linear independence in a couple of different ways:

1. A set of vectors are linearly independent if the only solution to  is .
2. The determinant of a matrix is 0 if and only if the columns are linearly dependent.

To use this method, we write the vectors as the columns of a matrix and then find the determinant.

Finding a basis for

Study Development Quickguide

## Spanning

When we say a set of vectors span , we mean that we can find any vector in by writing it as for scalars .

There is a very useful result that ‘ linearly independent vectors will span ’ so, if you are testing whether your vectors span , and you have linearly independent vectors, you know that they span .

## Showing we have a basis in one step

We may rewrite as , where is a matrix whose columns are . If is invertible, then are a basis for . The explanation of this is in the factsheet.

We also know that a matrix is invertible if and only if its determinant is not 0.

Therefore, to show that we have a basis:

Write the vectors as the columns of a matrix, find the determinant. If the determinant is 0, we do not have a basis. If the determinant is not 0, we have a basis.

## Finding a basis

Now that we know how to check if vectors are a basis, finding a basis is a pretty simple task. We know that we need to find the columns of an nxn matrix such that its determinant isn’t zero.

For example, if we are asked to find a basis for we can simply fill in the blanks of a 3x3 matrix. Start by choosing any vector in . We’ll call this Then, choose a second vector in - as long as this isn’t a scalar multiple of your first vector it will work. We write this as . Next, we write

Finding a basis for

Study Development Quickguide

We then find the determinant and choose values for , and such that the determinant is not 0.

If we are finding a basis for with , we have to check for linear independence each time we add a new vector past the second vector added.

**Hint:** there are vectors that we sometimes call the ‘standard basis vectors’. These are vectors that have only one non-zero entry that is 1. These are usually the easiest way to find a basis. For example, for , the ‘standard basis vectors’ are , and . If you are unsure, try selecting the first two basis vectors in this format, and it should make it much easier to find a third vector.

**Support**: Study Development offers workshops, short courses, 1 to 1 and small group tutorials.

* Join a tutorial or workshop on the [Study Development tutorial and workshop webpage](https://www.yorksj.ac.uk/students/study-skills/study-development-tutorials/) or search ‘YSJ study development tutorials.’
* Access our Study Success resources on the [Study Success webpage](https://www.yorksj.ac.uk/students/study-skills/study-success/) or search ‘YSJ study success.’