Vectors

Study Development Quick Guide

Operations

Addition and subtraction

We add vectors by adding entries in the same position to each other. For example:

$$\begin{pmatrix}1\\0\\4\end{pmatrix} + \begin{pmatrix}2\\5\\8\end{pmatrix} = \begin{pmatrix}1+2\\0+5\\4+8\end{pmatrix} = \begin{pmatrix}3\\5\\12\end{pmatrix}$$

Subtraction works similarly, for example:

$$\begin{pmatrix}10\\4\\6\end{pmatrix} - \begin{pmatrix}3\\2\\8\end{pmatrix} = \begin{pmatrix}10-3\\4-2\\6-8\end{pmatrix} = \begin{pmatrix}7\\2\\-2\end{pmatrix}$$

Like when we add numbers, we have that $u + -v = u - v$ for vectors u and v.

Scalar multiplication

When we multiply be a scalar (a number or variable that is not a vector) we simply multiply each entry in the vector by the scalar. For example:

$$3\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}3\times 2\\3\times 1\end{pmatrix} = \begin{pmatrix}6\\3\end{pmatrix}$$

Norm

The norm of a vector, $||v||$, represents the length of the vector. It is calculated with the formula:

$$\left\|\begin{pmatrix}v_1\\v_2\\\vdots\\v_n\end{pmatrix}\right\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

Dot product

The dot product of two vectors, also called the inner product or scalar product, tells you (informally) the amount that one vector goes in the same direction as the other vector.

The formula for the inner product is given by:

$\langle u, v\rangle = u \cdot v = u_1v_1 + u_2v_2 + \cdots + u_nv_n$

For example:

$\begin{pmatrix}10\\0\\4\end{pmatrix} \cdot \begin{pmatrix}2\\8\\3\end{pmatrix} = (10 \times 2) + (0 \times 8) + (4 \times 3) = 20 + 24 = 44$

Cross product

The cross product of two vectors finds a vector that is perpendicular to both of them. The formula to find the cross product between $u = \begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}$ and $v = \begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$ is:

$$\mathrm{u} \times v = \begin{pmatrix}u_2v_3 - u_3v_2\\u_3v_1 - u_1v_3\\u_1v_2 - u_2v_1\end{pmatrix}$$

The cross product is also given by

$$u \times v = \left||u|\right|\left||v|\right| sin(\theta)\, n$$

Where θ is the angle between u and v and n is a unit vector perpendicular to u and v.

Angle between two vectors

The formula for finding the angle θ between two vectors u and v is as follows:

$$\theta = cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$$

Orthogonality

If two vectors are perpendicular to each other, we call them orthogonal. If they are both of length 1, we call them orthonormal.

To test if they are orthogonal, we find their dot product. If this is 0, they are orthogonal, as when we calculate the angle between them, θ, we have $\theta = cos^{-1}(0) = 90°$ (or $\frac{\pi}{2}$ radians).

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Projection onto a vector

The formula for calculating the projection of u onto v ($\mathrm{proj}_v(u)$) is as follows:

$$\mathrm{proj}_v(u) = \frac{u \cdot v}{v \cdot v} v$$

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