Vectors

Study Development Worksheet

## Questions

1. Calculate:
	1. $\left(\begin{matrix}2\\1\\1\end{matrix}\right)+\left(\begin{matrix}1\\0\\3\end{matrix}\right)$
	2. $\left(\begin{matrix}1\\0\\-2\\4\end{matrix}\right)+\left(\begin{matrix}-1\\3\\1\\-3\end{matrix}\right)$
	3. $\left(\begin{matrix}3\\1\end{matrix}\right)-\left(\begin{matrix}1\\1\end{matrix}\right)$
	4. $\left(\begin{matrix}13\\12\\27\\2\end{matrix}\right)-\left(\begin{matrix}10\\-1\\30\\-10\end{matrix}\right)$
	5. $\left(\begin{matrix}1\\0\\3\end{matrix}\right)+\left(\begin{matrix}4\\1\\-2\end{matrix}\right)+\left(\begin{matrix}2\\-1\\4\end{matrix}\right)$
	6. $\left(\begin{matrix}1&3&-2\end{matrix}\right)+\left(\begin{matrix}-3&10&4\end{matrix}\right)$
	7. $\left(\begin{matrix}-1\\10\end{matrix}\right)-\left(\begin{matrix}3\\-4\end{matrix}\right)+\left(\begin{matrix}2\\11\end{matrix}\right)$
	8. $\left(\begin{matrix}\frac{1}{2}\\-1\\\frac{3}{2}\end{matrix}\right)+\frac{1}{2}\left(\begin{matrix}3\\-1\\4\end{matrix}\right)$
	9. $3\left(\begin{matrix}2\\1\\-2\end{matrix}\right)$
	10. $\frac{5}{4}\left(\begin{matrix}8\\-2\end{matrix}\right)$
	11. $-\left(\begin{matrix}2\\10\\-3\\2\end{matrix}\right)+3\left(\begin{matrix}-1\\3\\12\\1\end{matrix}\right)$

Vectors

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* 1. $\left|\left|\left(\begin{matrix}3\\2\\-1\\2\end{matrix}\right)\right|\right|$
	2. $\left|\left|\left(\begin{matrix}-4\\2\\1\end{matrix}\right)\right|\right|$
	3. $\left|\left|\left(\begin{matrix}5\\10\end{matrix}\right)\right|\right|+\left|\left|\left(\begin{matrix}-1\\3\end{matrix}\right)\right|\right|$
	4. $\left(\begin{matrix}-1\\-3\\0\end{matrix}\right)×\left(\begin{matrix}-3\\-2\\4\end{matrix}\right)$
	5. $\left(\begin{matrix}2\\-1\\7\end{matrix}\right)×\left(\begin{matrix}-1\\3\\12\end{matrix}\right)$
	6. $<\left(\begin{matrix}2\\1\end{matrix}\right),\left(\begin{matrix}3\\1\end{matrix}\right)>$
	7. $<\left(\begin{matrix}1\\3\\2\end{matrix}\right),\left(\begin{matrix}0\\-1\\3\end{matrix}\right)>$

Vectors

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1. Calculate the angle (in radians) between:
	1. $\left(\begin{matrix}1\\3\end{matrix}\right)$ and $\left(\begin{matrix}3\\-1\end{matrix}\right)$
	2. $\left(\begin{matrix}2\\1\\-3\end{matrix}\right)$ and $\left(\begin{matrix}-3\\1\\4\end{matrix}\right)$
	3. $\left(\begin{matrix}10\\3\\0\end{matrix}\right)$and $\left(\begin{matrix}-6\\20\\7\end{matrix}\right)$
	4. $\left(\begin{matrix}3\\10\end{matrix}\right)$ and $\left(\begin{matrix}-1\\2\end{matrix}\right)$
	5. $\frac{1}{3}\left(\begin{matrix}1\\-2\\2\end{matrix}\right)$ and $\frac{1}{\sqrt{73}}\left(\begin{matrix}-1\\6\\-6\end{matrix}\right)$
	6. $\left(\begin{matrix}7\\1\\0\\-4\end{matrix}\right)$ and $\left(\begin{matrix}-1\\3\\4\\-1\end{matrix}\right)$
	7. Which of these pairs of vectors are orthogonal?
	8. Which are orthonormal?
2. Find the projection of:
	1. $\left(\begin{matrix}1\\1\\4\end{matrix}\right)$ onto $\left(\begin{matrix}-1\\7\\3\end{matrix}\right)$
	2. $\left(\begin{matrix}3\\-1\end{matrix}\right)$ onto $\left(\begin{matrix}2\\7\end{matrix}\right)$
	3. $\left(\begin{matrix}0\\1\\3\\-2\end{matrix}\right)$ onto $\left(\begin{matrix}1\\3\\-1\\6\end{matrix}\right)$
3. Find
	1. The length of the vector $\left(\begin{matrix}2&-1&7&1\end{matrix}\right)$
	2. A vector that is perpendicular to the vectors $\left(\begin{matrix}10\\0\\1\end{matrix}\right)$ and $\left(\begin{matrix}7\\3\\1\end{matrix}\right)$.

Vectors

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## Answers

1. Calculate:
	1. $\left(\begin{matrix}2\\1\\1\end{matrix}\right)+\left(\begin{matrix}1\\0\\3\end{matrix}\right)$ = $\left(\begin{matrix}2+1\\1+0\\1+3\end{matrix}\right)=\left(\begin{matrix}3\\1\\4\end{matrix}\right)$
	2. $\left(\begin{matrix}0\\3\\-1\\1\end{matrix}\right)$
	3. $\left(\begin{matrix}2\\0\end{matrix}\right)$
	4. $\left(\begin{matrix}3\\13\\-3\\12\end{matrix}\right)$
	5. $\left(\begin{matrix}7\\0\\5\end{matrix}\right)$
	6. $\left(\begin{matrix}-2&13&2\end{matrix}\right)$
	7. $\left(\begin{matrix}-2\\25\end{matrix}\right)$
	8. $\left(\begin{matrix}2\\-\frac{3}{2}\\\frac{7}{2}\end{matrix}\right)$
	9. $\left(\begin{matrix}6\\3\\-6\end{matrix}\right)$
	10. $\left(\begin{matrix}10\\\frac{-5}{2}\end{matrix}\right)$
	11. $\left(\begin{matrix}-5\\-1\\39\\1\end{matrix}\right)$

Vectors

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* 1. $3\sqrt{2}$
	2. $\sqrt{21}$
	3. $5\sqrt{5}+\sqrt{10}$
	4. $\left(\begin{matrix}-12\\4\\-7\end{matrix}\right)$
	5. $\left(\begin{matrix}-33\\-17\\5\end{matrix}\right)$
	6. 7
	7. 3
1. Calculate the angle (in radians) between:
	1. $\left(\begin{matrix}1\\3\end{matrix}\right)$ and $\left(\begin{matrix}3\\-1\end{matrix}\right)$ $θ=cos^{-1}\left(\frac{<\left(\begin{matrix}1\\3\end{matrix}\right),\left(\begin{matrix}3\\-1\end{matrix}\right)>}{\left|\left(\begin{matrix}1\\3\end{matrix}\right)\right|\left|\left(\begin{matrix}3\\-1\end{matrix}\right)\right|}\right) $ = $cos^{-1}\left(\frac{0}{10}\right)$ = 0
	2. $cos^{-1}\left(\frac{-17}{2\sqrt{91}}\right)$ = $2.670^{c}$
	3. 0
	4. $0.755^{c}$
	5. 0
	6. 0
	7. a, c, e, f
	8. e

Vectors

Study Development Worksheet

1. Find the projection of:
	1. $proj\_{\left(\begin{matrix}-1&7&3\end{matrix}\right)}\left(\left(\begin{matrix}1\\1\\4\end{matrix}\right)\right)$ = $\frac{\left(\begin{matrix}1\\1\\4\end{matrix}\right)⋅\left(\begin{matrix}-1\\7\\3\end{matrix}\right)}{\left(\begin{matrix}-1\\7\\3\end{matrix}\right)⋅\left(\begin{matrix}-1\\7\\3\end{matrix}\right)}\left(\begin{matrix}-1\\7\\3\end{matrix}\right) $= $\frac{18}{21}\left(\begin{matrix}-1\\7\\3\end{matrix}\right)$ = $\frac{6}{7}\left(\begin{matrix}-1\\7\\3\end{matrix}\right)$
	2. $proj\_{\left(\begin{matrix}2&7\end{matrix}\right)}\left(\left(\begin{matrix}3\\-1\end{matrix}\right)\right)$ = $\frac{-1}{18}\left(\begin{matrix}2\\7\end{matrix}\right)$
	3. $proj\_{\left(\begin{matrix}1&3&-1&6\end{matrix}\right)}\left(\left(\begin{matrix}0\\1\\3\\-2\end{matrix}\right)\right)$ = $\frac{-12}{47}\left(\begin{matrix}1\\3\\-1\\6\end{matrix}\right)$
2. Find
	1. $\left|\left|\left(\begin{matrix}2&-1&7&1\end{matrix}\right)\right|\right|=\sqrt{2^{2}+\left(-1\right)^{2}+7^{2}+1^{2}}=\sqrt{4+1+49+1}=\sqrt{55}$
	2. A perpendicular vector is given by the cross product of the vectors.

$$\left(\begin{matrix}10\\0\\1\end{matrix}\right)×\left(\begin{matrix}7\\3\\1\end{matrix}\right)=\left(\begin{matrix}0×1-3×1\\7×1-10×1\\3×10-0×7\end{matrix}\right)=\left(\begin{matrix}-3\\-3\\30\end{matrix}\right)$$

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