The Law of Total Probability

Study Development Quick Guide

The (discrete) Law of Total Probability (also called the Law of Alternatives) states that if events $\{B\_{n} : n=1, 2,…\}$ are pairwise dijoint and their union covers the entire sample space, then

$$P\left(A\right)=\sum\_{n}^{}P\left(A∩B\_{n}\right)=\sum\_{n}^{}P\left(B\_{n}\right)P\left(B\_{n}\right)$$

(provided each $B\_{n}$ is measurable and $A$ exists in the same probability space).

## Steps:

1. Check if you can use the law of total probability. This means you need to decide whether $\{B\_{n} : n=1, 2,…\}$ are pairwise disjoint, measurable, their union covers the entire sample space, and if $A$ exists in the same probability space.
2. Write down your information. This will (usually) either be $P\left(A∩B\_{n}\right)$ for all $n$, or $P\left(B\_{n}\right)$ and $P\left(B\_{n}\right)$ for all $n.$ In some cases, you may be given $P(A)$ and asked to work out one of the other probabilities, in which case you will need to write down all the infromation you have, and then rearrange the equation after step 3.
3. Plug your information into one of the two equations, either $P\left(A\right)=\sum\_{n}^{}P\left(A∩B\_{n}\right)$, or $P\left(A\right)=\sum\_{n}^{}P\left(B\_{n}\right)P\left(B\_{n}\right)$.

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